

# Recomendación Basada en Grafos

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# Agenda Semestral

27 - 29 Oct	Active/Reinforcement Learning Recommender Systems	Gabriel della Maggiora y Javier Machin
3 - 5 Nov	Graph-based recommendation	Juan Pablo Salazar y Christopher Arenas
10 - 12 Nov	Applications: music	Miguel Fadic
17 - 19 Nov	Modelos graficos probabilisticos para sistemas recomendadores	Laura Cruz (invitada)

# Problema de Recomendación

- Nuevamente revisitamos el problema de recomendación
- Una alternativa válida a los métodos vistos hasta ahora es explotar las relaciones entre items en la forma de grafos.

# Hoy

- Associative retrieval techniques to alleviate the sparsity problem in CF (Huang et al. 2004)
- The link Prediction Problem for Social Networks (Liben-Nowel, Kleinberg, 2002)

# Paper 1

- Zan Huang, Hsinchun Chen, and Daniel Zeng. 2004. Applying associative retrieval techniques to alleviate the sparsity problem in collaborative filtering. ACM Trans. Inf. Syst. 22, 1 (January 2004), 116-142.

# Resumen

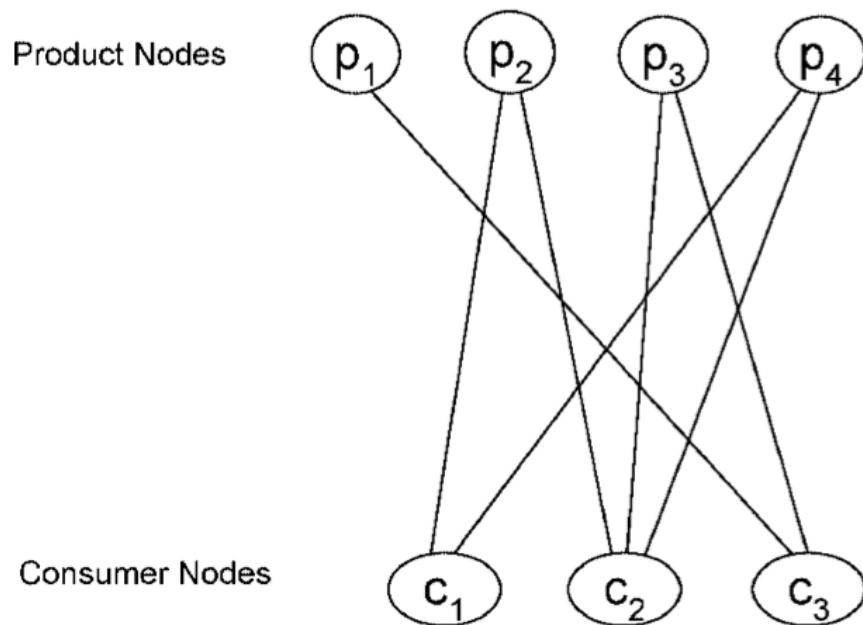
- Lidiar con el problema de escasez de evaluaciones del usuario (ratings)
- Filtrado Colaborativo es estudiado como un grafo bi-partito.
- Técnicas de recuperación asociativa son utilizadas sobre el grafo (Spreading Activation)
- RESULTADO: Cuando hay escasez de ratings, estas técnicas basadas en grafos mejoran los resultado del filtrado colaborativo

# El Problema de Escasez (Sparsity)

- Al 2004, los problemas cold-start y new-item se habían atacado usando:
  - Item-Base CF (Sarwar 2001)
  - Reducción de Dimensionalidad (Goldberg 2001)
  - Híbridos (Balanovic 2002, Basu 1998, Condliff 1999, etc.)
- Ninguno de los métodos mencionados había tenido consenso absoluto de su éxito

# CF como Recuperación Asociativa

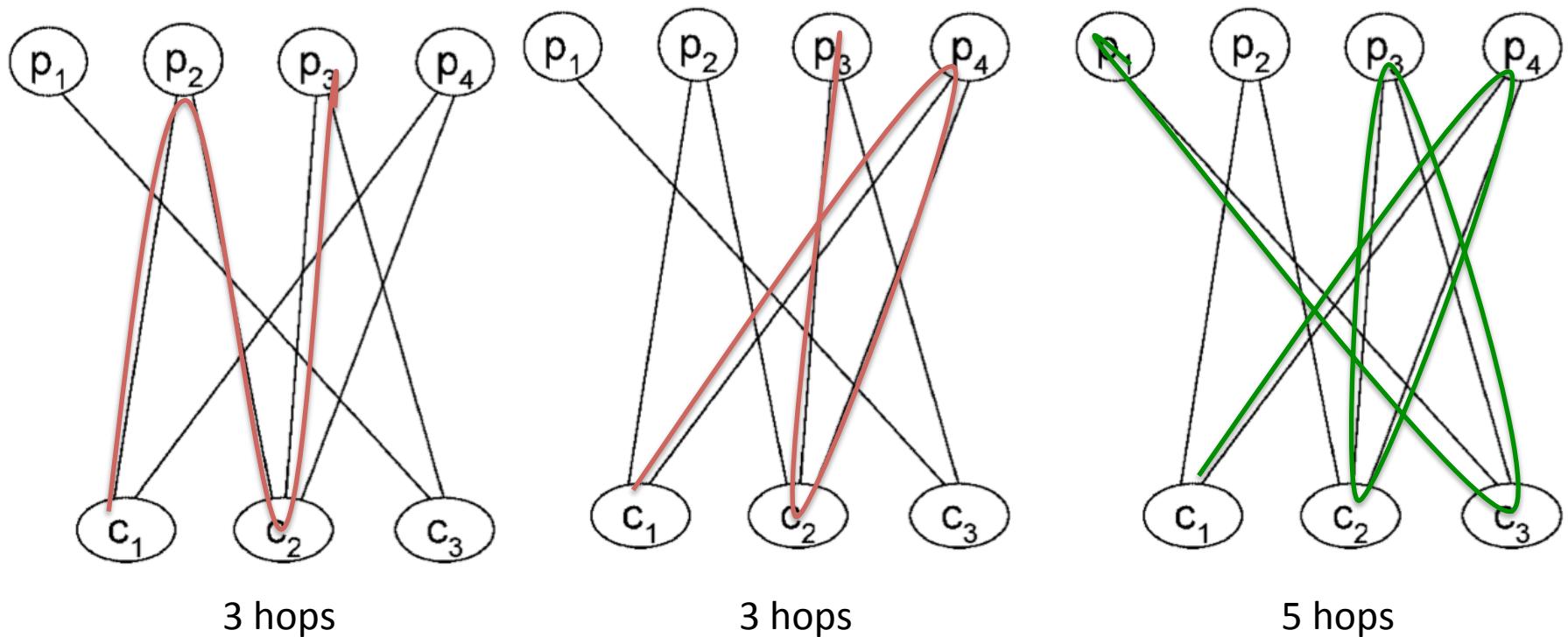
- Idea básica: construir un grafo entre usuarios e items y explorar asociaciones transitivas entre ellos.



	$p_1$	$p_2$	$p_3$	$p_4$
$c_1$	0	1	0	1
$c_2$	0	1	1	1
$c_3$	1	0	1	0

# CF como Recuperación Asociativa

- Idea básica: construir un grafo entre usuarios e items y explorar asociaciones transitivas entre ellos.



# Notación Matricial

- Consideremos la matriz consumidor/producto A
- Parámetros: M: hops,  $\alpha$ = decaimiento (peso asociado al enlace)

$$A_\alpha^M = \begin{cases} \alpha A, & \text{if } M = 1, \\ \alpha^2 A \cdot A^T \cdot A_\alpha^{M-2}, & \text{if } M = 3, 5, 7, \dots \end{cases}$$

# Ejemplo

- Dado A

$$\begin{matrix} & p_1 & p_2 & p_3 & p_4 \\ c_1 & \left[ \begin{matrix} 0 & 1 & 0 & 1 \end{matrix} \right] \\ c_2 & \left[ \begin{matrix} 0 & 1 & 1 & 1 \end{matrix} \right] \\ c_3 & \left[ \begin{matrix} 1 & 0 & 1 & 0 \end{matrix} \right] \end{matrix}$$

- Luego, para M = 3,  $\alpha = 0.5$

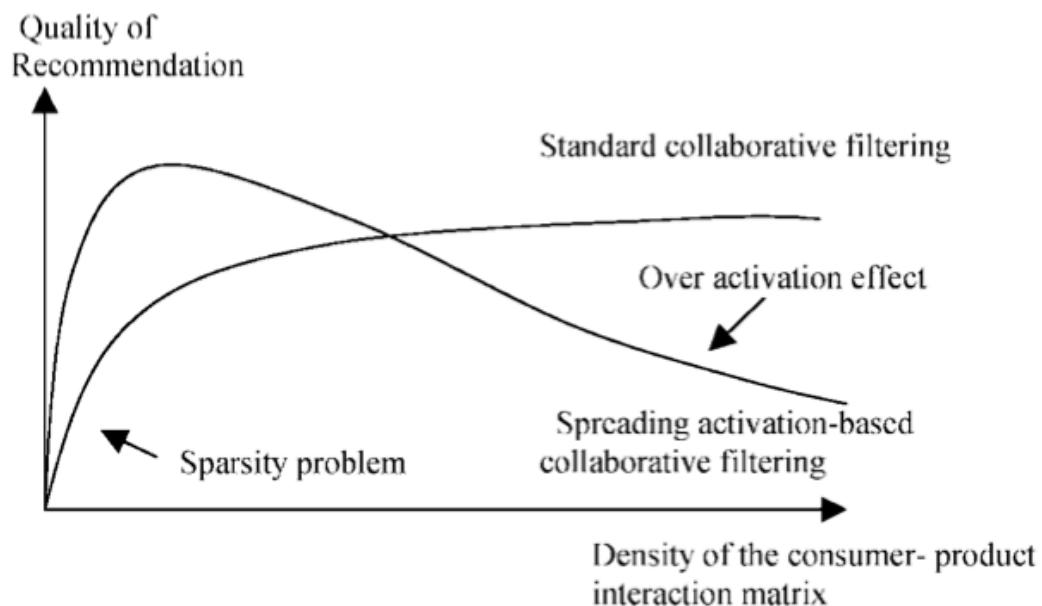
$$A_{0.5}^3 = \left[ \begin{matrix} 0 & 0.5 & 0.25 & 0.5 \\ 0.125 & 0.625 & 0.5 & 0.625 \\ 0.25 & 0.125 & 0.375 & 0.125 \end{matrix} \right]$$

- Luego, para M = 5,  $\alpha = 0.5$

$$A_{0.5}^5 = \left[ \begin{matrix} 0.0625 & 0.5625 & 0.375 & 0.5625 \\ 0.15625 & 0.75 & 0.59375 & 0.75 \\ 0.15625 & 0.21875 & 0.3125 & 0.21875 \end{matrix} \right]$$

# Supuesto de la Investigación

- Los métodos de Spreading Activation funcionarán mejor cuando la red tiene muy baja densidad, en caso contrario puede ocurrir sobre-activación.



# Modelos

- Constrained Leaky Capacitor Model (LCM)
- Branch-and-Bound
- Hopfield Net

# LCM

- Propuesto por Anderson (1983)

$$R(r \times r) = \begin{pmatrix} I(|P| \times |P|) & A^T(|C| \times |P|) \\ A(|P| \times |C|) & I(|C| \times |C|) \end{pmatrix}$$

- Pasos:

- Identificar nodo-vector inicial  $V$ , setear  $D(0)$
- Cálculo de nivel de activación

$$D(t) = V + M'D(t-1), \quad M = (1 - \gamma)I + \alpha R,$$

Donde  $(1-\gamma)$ : speed of decay (0.8),  $\alpha$ : efficiency (0.8)

- Condición de detención: en el paper = 10, top 50

# Branch-and-Bound

- Implementación basada en (Chen & Ng 1995)
- Paso 1, Inicialización: Nodo correspondiente al usuario es activado (1), los otros = 0. Cola  $Q_{\text{priority}}$  se inicializa con nodo usuario activo.
- Paso 2, Cálculo de activación: Sacar nodos de  $Q_{\text{priority}}$ , por cada nodo vecino calcular

$$\mu_j(t + 1) = \mu_i(t) \times t_{ij}$$

y agregar/actualizar nodo activado a  $Q_{\text{output}}$

- Paso 3, detención: determinada empíricamente (70)

# Holpfield Net

- Paralelo con red neuronal. Usuarios e items son neuronas. Sinapsis son las activaciones.
- Inicialización: igual que las anteriores
- Calculo de activación:

$$\mu_j(t+1) = f_s \left[ \sum_{i=0}^{n-1} t_{ij} \mu_i(t) \right], \quad 0 \leq j \leq n-1. \quad f_s(x) = \frac{1}{1 + \exp((\theta_1 - x)/\theta_2)}$$

- Condición de detención:

$$\sum_j \mu_j(t+1) - \sum_j \mu_j(t) < \varepsilon \times t.$$

# Estudio Experimental

- Tienda de libros en linea de China  
9,695 libros / 2,000 usuarios / 18,771 transacciones
- Métricas de evaluación:  
Precision, Recall, F-1
- Y utility rank

$$R_i = \sum_j \frac{p(i, j)}{2^{(j-1)/(h-1)}}$$

where  $p(i, j) = \begin{cases} 1, & \text{if item } j \text{ is in user } i's \text{ future purchase list,} \\ 0, & \text{otherwise.} \end{cases}$

$$R = 100 \frac{\sum_i R_i}{\sum_i R_i^{\max}},$$

# Resultados

Table I. Experimental Results for H1

Algorithm	Precision	Recall	F'-measure	Utility score
Hopfield	<b>0.0266</b>	<b>0.1519</b>	<b>0.0407</b>	<b>7.94</b>
3-hop	<b>0.0155</b>	0.0705	<b>0.0230</b>	<b>3.51</b>
User-based (Correlation)	0.0181	<b>0.1064</b>	0.0279	4.57
User-based (Vector Similarity)	0.0187	0.1089	0.0288	4.56
Item-based	0.0082	<b>0.0516</b>	0.0126	0.65

H1: Comparación de algoritmos bajo condiciones normales

Table II. Experimental Results for H2

Algorithm	Precision	Recall	F'-measure	Utility score
Hopfield	<b>0.0054</b>	<b>0.1122</b>	<b>0.0102</b>	<b>9.78</b>
3-hop	<b>0.0017</b>	0.0315	<b>0.0031</b>	<b>2.36</b>
User-based (Correlation)	0.0027	<b>0.0525</b>	0.0051	3.86
User-based (Vector Similarity)	0.0027	<b>0.0525</b>	0.0051	3.86
Item-based	0.0014	0.0282	0.0027	0.43

H1: Comparación de algoritmos con usuarios sparse

# Resultados 2

Table IV. Characteristics of the Graphs of Varying Degree of Sparsity

Graph	Number of links	Density	Average degree of customer node	Standard deviation of customer node degree	Average degree of book node	Standard deviation of book node degree
G1	4278	0.000031	1.607	4.378	0.124	0.422
G2	6382	0.000047	2.152	5.603	0.235	0.667
G3	9690	0.000071	3.011	7.182	0.409	1.069
G4	12952	0.000095	3.868	9.253	0.580	1.621
G5	16256	0.000119	4.732	11.106	0.750	2.095
G6	19376	0.000142	5.595	13.057	0.915	2.231
G7	21494	0.000157	6.189	14.569	1.026	2.321
G8	25526	0.000187	7.279	16.619	1.228	4.649
G9	28692	0.000210	8.120	17.831	1.386	4.921
G10	31826	0.000233	8.950	18.431	1.540	5.042
G11	35038	0.000256	9.805	19.358	1.700	5.143

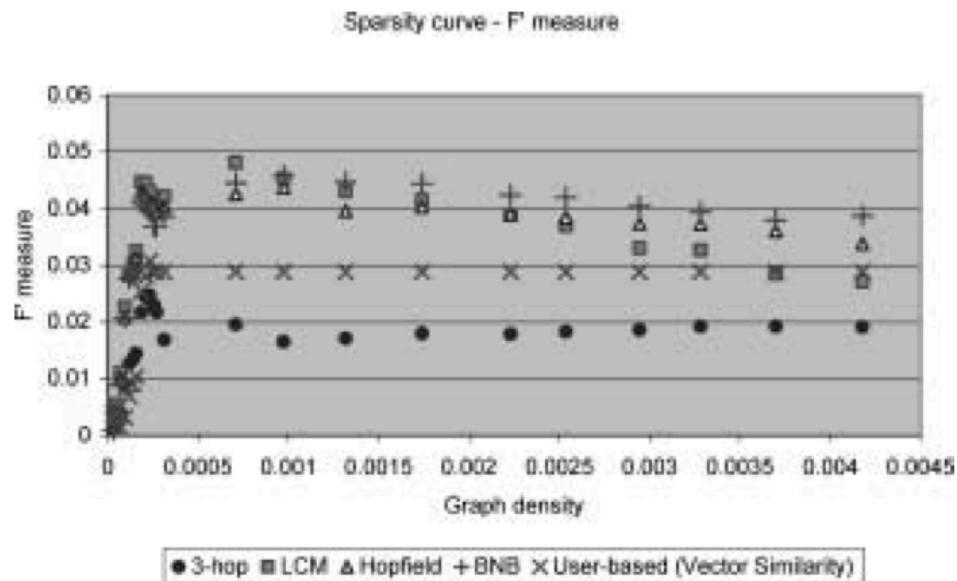


Fig. 4. Over-activation effect (with graphs enhanced by item associations).

# Lecciones

- H1, H2 y H3 se demuestran
- Sensibilidad de los parámetros:
  - LCM: no es muy sensible (alfa, gama e iteraciones)
  - BNB: diferencia en 70 y 100 iteraciones es baja, sobre 100 baja drásticamente
  - Hopfield Net: poca diferencia entre parámetros

# Paper 2

- Liben-Nowell, D., & Kleinberg, J. (2007). The link-prediction problem for social networks. *Journal of the American society for information science and technology*, 58(7), 1019-1031.

# El Problema

- ▶ **Link prediction problem:** Given the links in a social network at time  $t$  or during a time interval  $I$ , we wish to predict the links that will be added to the network during the later time interval from time  $t'$  to a some given future time.
- ▶ **Main approach:** Use measures of network-proximity adapted from graph theory, computer science, and the social sciences to determine which unconnected nodes are ‘close together’ in the topology of the network.

# Definiciones

$G = \langle V, E \rangle$  ← Social Network

$e = \langle u, v \rangle \in E$  ← Interaction between  $u$  and  $v$

$G[t_0, t_1]$  ← Given Subgraph as training set

$G[t_2, t_3]$  ← Infer new Edges/Used for testing

$\text{Score}(u, v)$  ← Likelihood that  $u$  and  $v$  share  
an edge (Proximity or Similarity)

Also, for a node  $x$ ,  $\Gamma(x)$  represents the set of neighbors of  $x$ .  $\text{degree}(x)$  is the size of the  $\Gamma(x)$ .

Imagen desde <http://be.amazd.com/link-prediction/>

# Notación para arXiv de Física

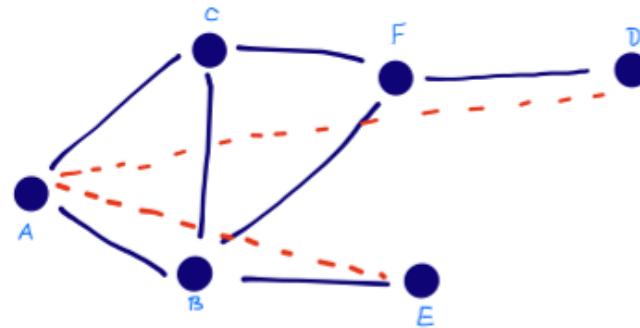
- ▶  $[t_0, t'_0]$  are the three years 1993 – 1996
- ▶  $[t_1, t'_1]$  are the three years 1997 – 1999
- ▶  $G[1993, 1996] = G_{collab} = \langle A, E_{old} \rangle$
- ▶  $E_{new}$  is the set of edges  $\langle u, v \rangle$  such that authors  $u$  and  $v$  co-authored an article sometime during 1997 – 1999 but not during 1993 – 1996
- ▶ Each link predictor  $p$  outputs a ranked list  $L_p$  of pairs in  $A \times A - E_{old}$ . List is ordered according to decreasing values of  $\text{score}(x, y)$  for  $\langle x, y \rangle \in A \times A - E_{old}$

# Métricas 1: distancia en el grafo

*negated*

$$\text{Score}(x,y) = \sqrt{\text{Length of Shortest Path Between } x \text{ and } y}$$

The measure follows the notion that social networks are **small worlds**, in which individuals are related through short chains.



$$\text{Score}(A,E) = -2$$

$$\text{Score}(A,D) = -3$$

desc order

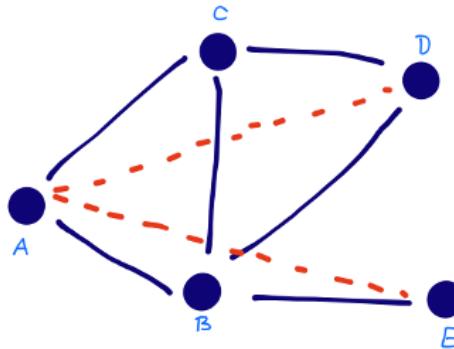
The use of negated (instead of original) shortest-path distance ensures that the proximity measure  $GD(x,y)$  increases as  $x$  and  $y$  get closer.

Imagen desde <http://be.amzd.com/link-prediction/>

# Vecinos en Común

$$\text{Score}(x, y) = |\Gamma(x) \cap \Gamma(y)|$$

Neighbors of x  
list comparison :  $O(V \cdot V \log V)$



$$|\Gamma(A) \cap \Gamma(D)|$$

$\downarrow$   
 $B, C$   
 $\downarrow$   
 $B, C$

$S = 2 \checkmark$

$$|\Gamma(A) \cap \Gamma(E)|$$

$\downarrow$   
 $B, C$   
 $\downarrow$   
 $B$

$S = 1$

*triadic closure*

# Jaccard

$$\text{Score}(x, y) = \frac{|\Gamma(x) \cap \Gamma(y)|}{|\Gamma(x) \cup \Gamma(y)|}$$

*Common friends*      ←  
*Total friends*      ←

*This metric solves the problem where two nodes could have many common neighbors because they have lots of neighbors, not because they are strongly related*

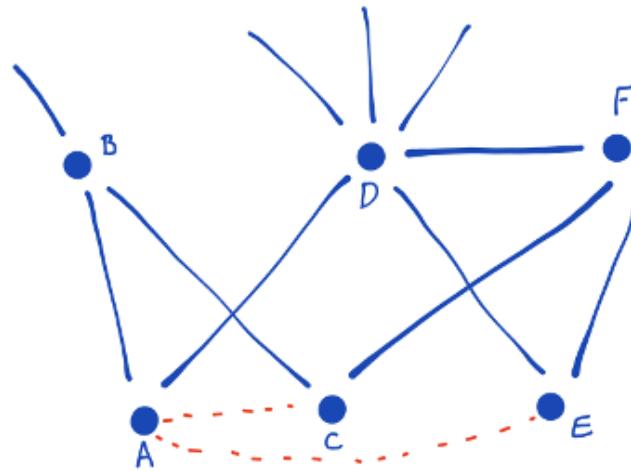
# Adamic-Adar

$$\text{Score}(x, y) = \sum_{z \in \Gamma(x) \cap \Gamma(y)} \frac{1}{\log|\Gamma(z)|}$$

Frequency of  $Z$

Weighting rare  
features more heavily

list comparison :  $O(V \cdot V \log V)$



$$\Gamma(A) \cap \Gamma(C) = B$$

$$\Gamma(A) \cap \Gamma(E) = D$$

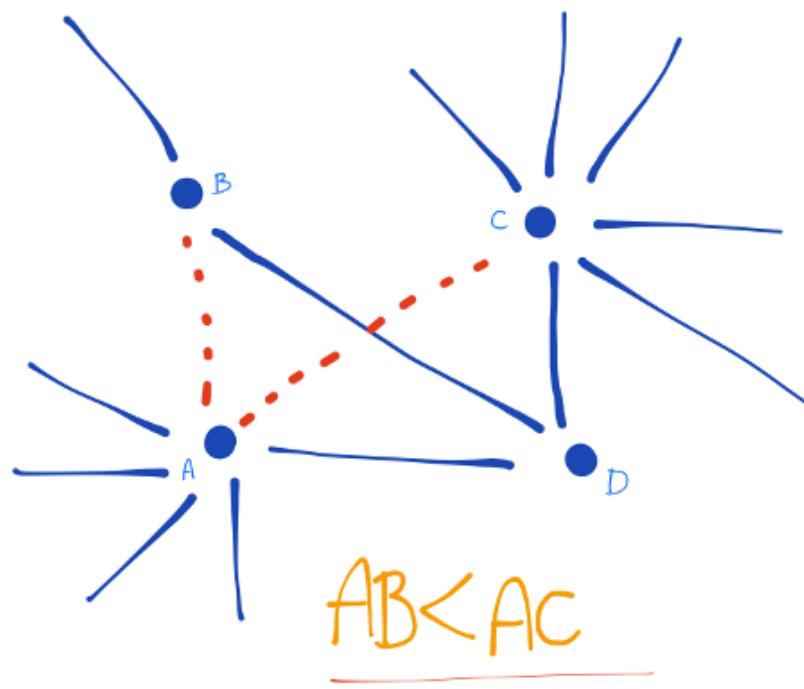
$$\frac{1}{\log(\Gamma(B))} = \frac{1}{\log 3} \approx \underline{\underline{2.09}}$$

triadic closure

$$\frac{1}{\log(\Gamma(D))} = \frac{1}{\log 6} \approx 1.2$$

# Preferential Attachment

$$\text{Score}(x,y) = |\Gamma(x)| \cdot |\Gamma(y)|$$



*The link between A and C is more probable than the link between A and B  
as C have many more neighbors than B*

# Katz

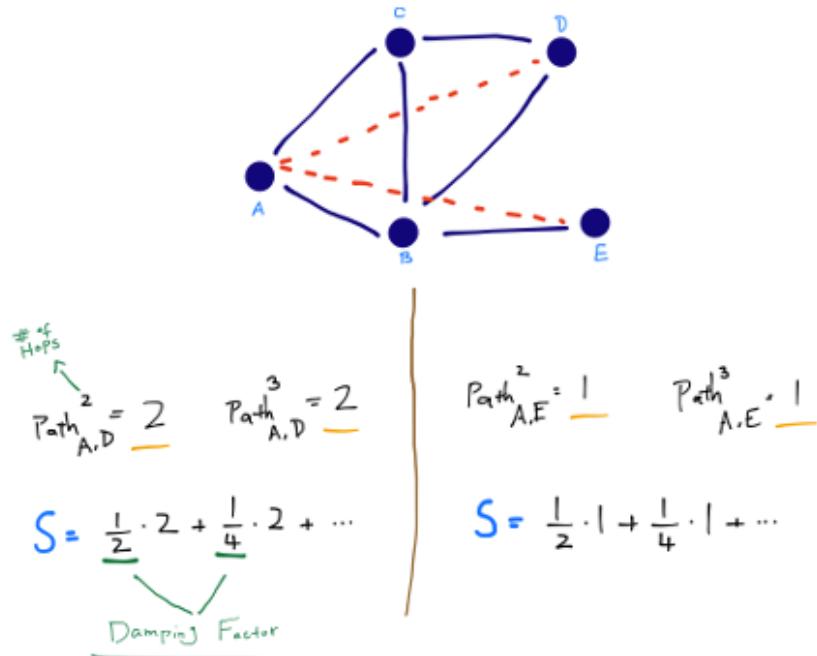
$$\text{Score}(x, y) = \sum_{L=1}^{\infty} \beta^L \cdot |\text{Path}_{x,y}^L|$$

↓

Set of all length L  
Paths from x to y

exponentially damped  
by length

A very small  $\beta$  yields predictions much like common neighbors, because paths of length three or more contribute very little to the summation.



# Espectrales/Random Walk

- Hitting Time

- Hitting Time
  - ▶ Consider a *random walk* on  $G_{collab}$  which starts at  $x$  and iteratively moves to a neighbour of  $x$  chosen uniformly at random from  $\Gamma(x)$ .
  - ▶ The **Hitting Time**  $H_{x,y}$  from  $x$  to  $y$  is the expected number of steps it takes for the RW starting at  $x$  to reach  $y$ .

$$\text{score}(x, y) = -H_{x,y}$$

- Rooted Page Rank

- Rooted PageRank:

$$\text{score}(x, y) = \text{stationary distribution weight of } y \text{ under this scheme}$$

- SimRank

- SimRank <sub>$\gamma$</sub> : Let  $\text{similarity}(x, y)$  be a fixed point of

$$\text{similarity}(x, y) = \gamma \frac{\sum_{a \in \Gamma(x)} \sum_{b \in \Gamma(y)} \text{similarity}(a, b)}{|\Gamma(x)| |\Gamma(y)|}$$

where  $\gamma \in [0, 1]$

$$\text{score}(x, y) = \text{similarity}(x, y)$$

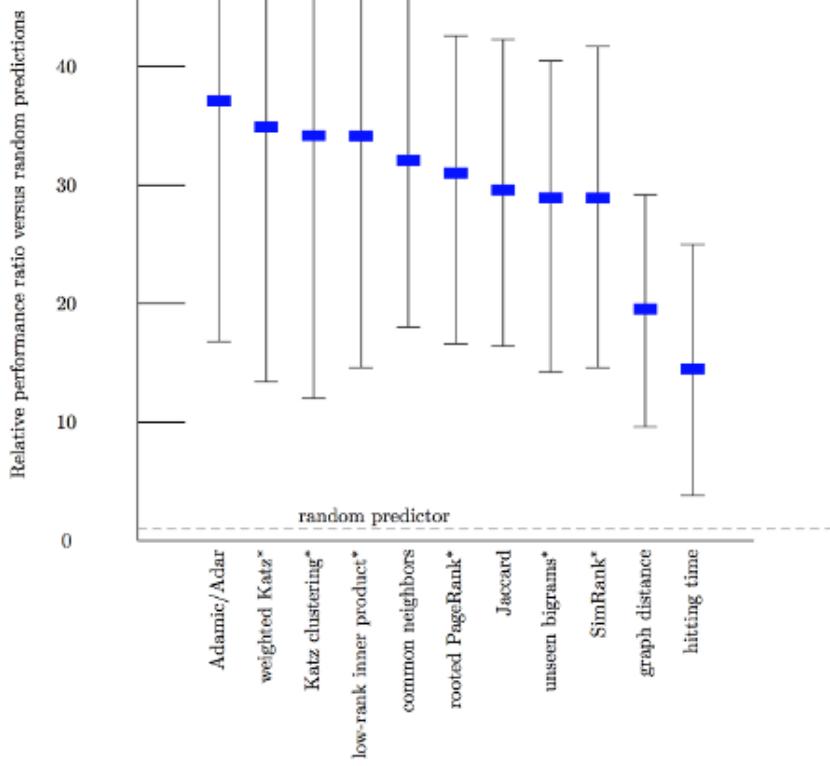
# Resultados

- ▶ Prediction accuracy will be tabulated in terms of relative improvement over a *random predictor*
- ▶ The random predictor simply predicts randomly selected pairs of authors from Core who did not collaborate during the training interval 1993 – 1996.
- ▶ The probability the random prediction is correct is

$$\frac{1}{\binom{|Core|}{2} - |E_{old}|}$$

- ▶ This value ranges from 0.15% in cond-mat to 0.48% in astro-ph

# Resultados 2



The number on the left is the number of factor of improvements over the random prediction. i.e. the Adamic/Adar measure is **about 37 times more accurate** than the random predictor

predictor	astro-ph	cond-mat	gr-qc	hep-ph	hep-th
probability that a random prediction is correct	0.475%	0.147%	0.341%	0.207%	0.153%
graph distance (all distance-two pairs)	<i>9.6</i>	<i>25.3</i>	<i>21.4</i>	<i>12.2</i>	<i>29.2</i>
common neighbors	<b>18.0</b>	<b>41.1</b>	<b>27.2</b>	<b>27.0</b>	<b>47.2</b>
preferential attachment	4.7	6.1	7.6	15.2	7.5
Adamic/Adar	<i>16.8</i>	<b>54.8</b>	<b>30.1</b>	<b>33.3</b>	<b>50.5</b>
Jaccard	<i>16.4</i>	<b>42.3</b>	19.9	<b>27.7</b>	<i>41.7</i>
SimRank	$\gamma = 0.8$	<i>14.6</i>	<i>39.3</i>	<i>22.8</i>	<i>26.1</i>
hitting time	6.5	23.8	<i>25.0</i>	3.8	13.4
hitting time, stationary-distribution normed	5.3	23.8	11.0	11.3	21.3
commute time	5.2	15.5	<b>33.1</b>	<b>17.1</b>	23.4
commute time, stationary-distribution normed	5.3	16.1	11.0	11.3	16.3
rooted PageRank	$\alpha = 0.01$	<i>10.8</i>	<b>28.0</b>	<b>33.1</b>	<i>18.7</i>
	$\alpha = 0.05$	<i>13.8</i>	<i>39.9</i>	<b>35.3</b>	<i>24.6</i>
	$\alpha = 0.15$	<i>16.6</i>	<b>41.1</b>	<b>27.2</b>	<i>27.6</i>
	$\alpha = 0.30$	<i>17.1</i>	<b>42.3</b>	<i>25.0</i>	<b>29.9</b>
	$\alpha = 0.50$	<i>16.8</i>	<b>41.1</b>	<i>24.3</i>	<b>30.7</b>
Katz (weighted)	$\beta = 0.05$	3.0	21.4	19.9	2.4
	$\beta = 0.005$	<i>13.4</i>	<b>54.8</b>	<b>30.1</b>	<i>24.0</i>
	$\beta = 0.0005$	<i>14.5</i>	<b>54.2</b>	<b>30.1</b>	<i>32.6</i>
Katz (unweighted)	$\beta = 0.05$	<i>10.9</i>	<b>41.7</b>	<b>37.5</b>	<i>18.7</i>
	$\beta = 0.005$	<i>16.8</i>	<b>41.7</b>	<b>37.5</b>	<i>24.2</i>
	$\beta = 0.0005$	<i>16.8</i>	<b>41.7</b>	<b>37.5</b>	<i>24.9</i>

Figure 3-3: Performance of the basic predictors on the link-prediction task defined in Section 3.2. See Sections 3.3.1, 3.3.2, and 3.3.3 for definitions of these predictors. For each predictor and each arXiv section, the displayed number specifies the factor improvement over random prediction. Two predictors in particular are used as baselines for comparison: graph distance and common neighbors. Italicized entries have performance at least as good as the graph-distance predictor; bold entries are at least as good as the common-neighbors predictor. See also Figure 3-4.

Chart showing the numerical results on multiple sections of the arXiv coauthorship network. Different sections of arXiv yield different results.

# Referencias

- Zan Huang, Hsinchun Chen, and Daniel Zeng. 2004. Applying associative retrieval techniques to alleviate the sparsity problem in collaborative filtering. *ACM Trans. Inf. Syst.* 22, 1 (January 2004), 116-142.
- Liben-Nowell, D., & Kleinberg, J. (2007). The link-prediction problem for social networks. *Journal of the American society for information science and technology*, 58(7), 1019-1031.
- G. Jeh and J. Widom. SimRank: A measure of structural-context similarity. In Proceedings of the Eighth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Edmonton, Alberta, Canada, July 2002.
- Nguyen, P., Tomeo, P., Di Noia, T., & Di Sciascio, E. (2015, May). An evaluation of SimRank and Personalized PageRank to build a recommender system for the Web of Data. In Proceedings of the 24th International Conference on World Wide Web Companion (pp. 1477-1482). International World Wide Web Conferences Steering Committee.